INTEGRATION BEE 2025 WRITTEN EXAM

CLEARLY write your first and last name at the top of this page. Otherwise, on this page you should write only your final answers next to the corresponding problems and box them. There is no penalty for an incorrect answer. Unless otherwise specified, assume natural domain restrictions; no need to include them in your answer. Answers must be given in CLOSED FORM (no infinite series)! +C is not necessary.

The last problem is a tie-breaker. You must answer as a decimal (xx.xxx ...). Your score for the last problem will be $e^{-|A-I|}$ where A is your answer and I is the actual value of the integral. Good luck!

(1) $$\int e^{2ix}\cos(x)\,dx = \sin(x) - \frac{2}{3}\sin^3(x) - \frac{2}{3}i\cos^3(x)$$

(2) $$\int 2025^{2025^x+x}\,dx = \frac{2025^{2025^x}}{\ln^2(2025)}$$

(3) $$\int_0^4 x\sqrt{4-x}\,dx = \frac{128}{15}$$

(4) $$\int_0^{2\pi} \max(\sin(x),\cos(x))\,dx = 2\sqrt{2}$$

(5) $$\int \frac{1}{(x-1)(x-2)(x-3)(x-4)}\,dx = \frac{1}{6}(-\ln(x-1) + 3\ln(x-2) - 3\ln(x-3) + \ln(x-4))$$

(6) $$\int_0^{\pi} \sin^4(x)\cos^2(x)\,dx = \frac{\pi}{16}$$

(7) $$\int \frac{\sin(x)+\cos(x)}{2\sin(x)+3\cos(x)}\,dx = \frac{1}{13}(5x - \ln(2\sin(x)+3\cos(x))$$

(8) $$\int \frac{e^x(x-3)}{(x+1)^5}\,dx = \frac{e^x}{(x+1)^4}$$

(9) $$\int \sec^6(x) - \tan^6(x)\,dx = x + \tan^3(x)$$

(10) $$\int_1^{2025} \sqrt{1+\frac{1}{x^2}+\frac{1}{(x+1)^2}}\,dx = 2024 + \log\left(\frac{2025}{1013}\right)$$

(11) $$\int \sum_{n=0}^{\infty}\left(\frac{x^n}{(2n)!} + \frac{x^{2n}}{n!}\right)dx = \frac{e^x + e^{2x} - e^{-x}}{2}$$

(12) $$\int_0^1 \lfloor \log_2\left(\frac{1}{x}\right)\rfloor\,dx = 1$$

(13) $$\int_0^{\frac{\pi}{4}} \frac{1}{2\tan(x) + \frac{1}{2\tan(x)+\frac{1}{2\tan(x)+\dots}}}\,dx = \ln\left(1+\frac{1}{\sqrt{2}}\right)$$

(14) $$\int_0^1 \frac{\sqrt{x}-\sqrt[4]{x}}{\ln(x)}\,dx = \ln\left(\frac{6}{5}\right)$$

(15) $$\int_0^\infty e^{-ax}\cos(x)\,dx = \frac{a}{1+a^2}$$

(16) $$\int \frac{\ln(x)}{x}\cdot\frac{1}{x^{\ln(x)}+x^{-\ln(x)}} = \frac{1}{2}\arctan(x^{\ln(x)})$$

(17) $$\int_{-\infty}^\infty (20x+25)e^{-2025x^2}\,dx = \frac{5\sqrt{\pi}}{9}$$

(18) $$\lim_{n\to\infty}\sqrt{n}\int_0^2 [x(2-x)]^n\,dx = \sqrt{\pi}$$

(19) Let $f(n) = \begin{cases} 1 & n \text{ has an even number of 1 when written in base 3} \\ -1 & \text{otherwise} \end{cases}$

$$\int_0^1 \sum_{n=0}^\infty f(n)x^n = \ln(2)$$

(20) $$\frac{\int_0^\infty (1+x^2)^{-2024}\,dx}{\int_0^\infty (1+x^2)^{-2025}\,dx} = \frac{4048}{4047}$$

(21) $$\int \frac{xe^x - e^x}{x^2+e^{2x}}\,dx = \arctan\left(\frac{e^x}{x}\right)$$

(22) 2025 has the property that $(20+25)^2 = 2025$.

There are 2 other 4-digit numbers that have this property. Let a, b be the other 2 with $a < b$.

$$\int_a^b dx = 6776$$

(23) $$\int_{-\infty}^\infty \frac{1}{(x+\frac{1}{x}-1)^2}\,dx = \frac{4\pi}{3\sqrt{3}}$$

(24) $$\lim_{n\to\infty}\int_0^\infty \frac{n\cos(x)}{n^2x^2+1}\,dx = \frac{\pi}{2}$$

(25) $$\int_0^1 e^{e^x}\,dx \approx 6.31656$$