INTEGRATION BEE 2024 WRITTEN EXAM SOLUTION MANUAL

CLEARLY write your first and last name at the top of this page. Otherwise, on this page you should write only your final answers next to the corresponding problems and box them. There is no penalty for an incorrect answer. Unless otherwise specified, assume natural domain restrictions; no need to include them in your answer. Answers must be given in CLOSED FORM (no infinite series)! +C is not necessary.

(1) $$UM \int_0^1 S\,dx = UMS$$

(2) $$\int \left(x - \frac{1}{x}\right)^2 dx = \frac{x^3}{3} - 2x - \frac{1}{x} + C$$

(3) $$\int \cot(x) - \frac{1}{\cot(x)} dx = \ln|\sin(x)| + \ln|\cos(x)| + C$$

(4) $$\int e^{5x}\sin(2x)\,dx = \frac{1}{29}[5e^{5x}\sin(2x) - 2e^{5x}\cos(2x)] + C$$

(5) $$\int (1-\sec(x))^2\,dx = -2\ln|\tan(x)+\sec(x)| + \tan(x) + x + C$$

(6) $$\int x^2 e^{-x}\,dx = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$$

(7) $$\int \frac{\cos(x)-3}{\sin(x)-3x}\,dx = \ln(|\sin(x)-3x|) + C$$

(8) $$\int x\sqrt{x\sqrt{x\sqrt{x\sqrt{\cdots}}}}dx = \frac{1}{3}x^3 + C$$

(9) $$\int_8^{63} \frac{1}{\sqrt{1+\sqrt{1+x}}} dx = \frac{64}{3}$$

(10) $$\int \frac{1}{e^x - 1} dx = \ln(1 - e^{-x}) + C$$

(11) $$\lim_{n\to\infty} \int_0^\infty \frac{1}{(1+x/n)^n} dx = 1$$

(12) $$\int_{-1}^1 -\left(\frac{x^3}{3} - x\right)\sin(\pi x)\,dx = \frac{4}{\pi^3} + \frac{4}{3\pi}$$

(13) $$\int_0^1 \frac{1-x^7}{1-x} dx = \sum_{k=1}^{7} \frac{1}{k} = \frac{363}{140}$$

(14) $$\int_4^\infty \frac{2x-1}{(x-1)(x-2)(x-3)}\,dx = \ln\left(\frac{8}{\sqrt{3}}\right)$$

(15) $$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{1}{2}\left(\frac{x}{1+x^2}+\tan^{-1}(x)\right)$$

(16) $$\int_0^{\pi} \frac{x\sin^4(x)}{\sin^4(x)+\cos^4(x)}\,dx = \frac{\pi^2}{4}$$

(17) $$\int_0^{\infty} \frac{x^4-2x+1}{x^8-1} = 0$$

(18) $$\int \frac{\sin(20x)+\sin(24x)}{\cos(20x)+\cos(24x)}\,dx = \frac{\ln(\cos(22x))}{22}$$

(19) $$\int \frac{1}{1-\cosh(ax)}\,dx = \frac{1}{a}\coth\left(\frac{ax}{2}\right)+C$$

(20) $$\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{1+\cos(x)}{x+\sin(x)}+\frac{x+\sin(x)}{1+\cos(x)} = \frac{5\pi}{3\sqrt{3}}+\ln\left(\frac{3\sqrt{3}+4\pi}{3\sqrt{3}+2\pi}\right)$$

(21) $$\int \frac{2\sin(2x)+2x\cos(2x)(x+2)}{(x+2)^2}dx = \frac{x\sin(2x)}{x+2}+C$$

(22) $$\int_0^1 \ln(x)\sin(\ln(x))dx = \frac{1}{2}$$

(23) $$\int \frac{1}{x^2+x+1}\,dx = \frac{2}{\sqrt{3}}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+C$$

(24) $$\int_0^{\infty} \sin(x^2+x)+\sin(x^2-x)dx = \sqrt{\frac{\pi}{2}}\left(\cos(\frac{1}{4})-\sin(\frac{1}{4})\right)$$

(25) $$\int_0^{\infty} \frac{\arctan(\frac{2x}{x^2+1})}{x}dx = \frac{\pi}{2}\ln(3+2\sqrt{2})$$

(26) $$\int_0^{\infty} \frac{\ln(x)\ln(1+x)}{1+x^2}dx = \frac{\pi^3}{16}$$

f turns all the 1s into 01s when written in binary. Ex: $f(0.101) = 0.01001$

(27) $$\int_0^1 f(x) = \frac{1}{5}$$

(28) $$\int_0^{\infty} \frac{\ln(x)}{1+x^4} = -\frac{\pi^2}{8\sqrt{2}}$$

(29) $$\int_0^{\infty} \frac{x^4}{1+x^8}\,dx = \frac{\pi}{8}\sec(\frac{\pi}{8})$$

(30) $$\int_0^{\infty} e^{-x^2}\sin\left(\frac{1}{x^2}\right) = \frac{\sqrt{\pi}}{2}e^{-\sqrt{2}}\cos(\sqrt{2})$$